Partial Information*

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No formal system can be a satisfactory vehicle for natural language interpretation unless it allows for some degree of underdefinedness. We are finite beings, our capacities for perceiving our surroundings are limited and since the world of phenomena is immensely large this means we can perceive only part of the world. We see, feel and hear parts of reality, not the whole of it, and it seems that a sentence containing a verb of perception like ‘John sees a house burn’ is most naturally treated as saying that the subject sees an incomplete world in which the embedded sentence is true (see Barwise (1981) for this analysis). But if we want to analyse perception verbs thus, we must introduce some form of incompleteness into our formal system, the system must be able to deal with partial information.

This is one reason for ‘going partial’, a second (but related) reason is the coarse-grainedness of traditional theories of meaning. Theories based on standard logic conflate the meanings of sentences that are classically equivalent, even if these sentences are not strictly synonymous. Here is an example.

(1) John is walking

(2) John is walking and Bill is talking or not talking

According to our intuition these are not synonymous sentences in the strictest sense, the first one does not even mention Bill or his talking, while the second one does. But no theory based on classical logic will be able to discriminate between the two. In possible worlds semantics, for example, the meaning of (1) will be a certain set of possible worlds (the set of worlds in which John walks) and the meaning of (2) will be the intersection of this set with the set that is the meaning of ‘Bill is talking or not talking’. But the latter sentence

is a classical tautology and so its meaning will be the set of all possible worlds. Hence (1) and (2) are predicted to be synonymous, which strictly speaking they are not.

At first blush it might seem that this incongruence between theoretical prediction and observed fact is not that important. The standard possible worlds theory, after all, gives a correct prediction about the relation of entailment here, even if it doesn’t predict our intuitions about strict synonymy accurately. The two sentences entail each other, if one is true the other is, and this fact correctly follows from the possible worlds approach. We see here that the relation of strict synonymy is somewhat more fine-grained (somewhat stronger) than the relation of co-entailment is. Cannot we take the position that, although this is true, the latter relation is a good enough approximation of the former and that the classical theory, since it gets the facts about entailment right, will suffice as an approximation to the theory of synonymy in natural language?

Unfortunately, such a strategy cannot work. Since the meaning of (1) does not strictly coincide with that of (2) the two propositions will have different properties. This in itself would be unproblematic if natural language were not rich enough to express these differences. But it is, and we can see this when we embed the sentences in a context of propositional attitude as is done in (3) and (4) below. Since (1) and (2) get the same semantical value in the classical system, sentences (3) and (4) are treated as being equivalent too. But Mary may believe (1) without believing anything about Bill at all and so, in particular, without believing (2); that is (3) may be true while (4) is false.

3) Mary believes that John is walking

4) Mary believes that John is walking and Bill is talking or not talking

So, here we are faced with a case where standard possible worlds semantics does not only give a wrong prediction about the notion of strict synonymy but where it also fails to account for the natural relation of logical consequence. Similarly, since (1) and (2) are treated as equivalent, by Compositionality (5) and (6) must be too.

5) Mary sees John walk

6) Mary sees John walk and Bill talk or not talk
But (6) entails that Mary sees Bill, while (5) clearly does not. Again, a wrong prediction about strict synonymy leads to a wrong prediction about entailment. We cannot content ourselves with an imperfect approximation of the relation of synonymy, since such an imperfection will immediately and necessarily lead to further imperfections in the way the relation of logical consequence is treated.

The introduction of partiality helps out here. If we allow ourselves to evaluate sentences on the basis of partial information only, i.e. allow them to be neither true nor false in some cases, then (1) and (2) will no longer be equivalent and neither will the two pairs (3) and (4) and (5) and (6) be. For example, if Mary does not see Bill at all, then the sentence ‘Bill talks’ will be undefined in the part of the world that is seen by her and as a consequence we may take it that ‘Bill talks or doesn’t talk’ and (2) are both undefined in that situation as well. Of course, (1) may still be true in that situation and so we find that (2) no longer follows from (1). Thus the introduction of partiality leads to a more fine-grained notion of entailment. Co-entailment in a theory based on partial information will be a better approximation to synonymy in natural language than classical co-entailment is.

How can we set up a logic that is based on partial information? In classical logic a sentence is either true or false, but now we must allow for a truth value gap, the possibility that a sentence has no truth value at all. So a sentence may be true and not false, false and not true, or neither true nor false. Let us call these three possibilities truth combinations. If we want to define a partial logic we need to specify how the truth combination of a complex sentence depends on the truth combinations of its parts. We can do this for the language of classical propositional logic in a particularly elegant way. Truth and falsity can be computed just as it is done ordinarily, provided that truth conditions and falsity conditions are separated:

(7) a. $\neg \varphi$ is true if and only if $\varphi$ is false
   b. $\neg \varphi$ is false if and only if $\varphi$ is true

(8) a. $\varphi \land \psi$ is true if and only if $\varphi$ is true and $\psi$ is true
   b. $\varphi \land \psi$ is false if and only if $\varphi$ is false or $\psi$ is false

(9) a. $\varphi \lor \psi$ is true if and only if $\varphi$ is true or $\psi$ is true
   b. $\varphi \lor \psi$ is false if and only if $\varphi$ is false and $\psi$ is false
So, for example, if \( \varphi \) is neither true nor false and \( \psi \) is true, then \( \varphi \land \psi \) is neither true nor false as well: it is not true by (8a) since \( \varphi \) is not and it is not false by (8b) since neither \( \varphi \) nor \( \psi \) is. Reasoning similarly in all other cases we obtain the following tables for the classical connectives (where \( T \) stands for truth, \( F \) for falsity and \( N \) for the lack of both):

\[
\begin{array}{ccc|ccc|ccc}
\land & T & F & N & \lor & T & F & N & \neg & T & F \\
T & T & F & N & T & T & T & T & T & F \\
F & F & F & F & F & T & F & N & F & T \\
\end{array}
\]

The truth tables that we obtain in this way are called the (strong) Kleene tables, after Kleene (1952). The reader may note that if sentences (1) and (2) are interpreted according to these tables they are no longer equivalent.

The logic that will be obtained in this way will be a partial propositional logic and will clearly not be rich enough to be an adequate tool for the analysis of natural language. But it turns out that richer logics can be defined on the basis of these ideas as well. It is not difficult to define partial predicate logics, partial modal logics and even partial typed logics on the basis of the strong Kleene tables. These logics generalise their ordinary variants, have virtually the same syntax as their ordinary variants, and allow us to pay heed to the fact that the information that supports our perceptions and beliefs is partial and not total.

See also: coreference; perception verbs; propositional attitudes.

References


