1) Suppose we have $N$ non-interacting bosons with two single particle states with energies $E_1 = 0$ and $E_2 = \varepsilon > 0$. Find an expression for the partition function. What is the probability that $n$ particles will be found in the excited state? Find the mean energy, the free energy and the entropy. Compare these results to a system with $N$ distinguishable particles but with otherwise the same available states.

2) Suppose we have $N$ noninteracting fermions with $2N$ single particle states, where the first $N$ states have energies $E_1 = 0$ and the second $N$ have energies $E_2 = \varepsilon$. Write down an expression for the partition function. Assuming high temperatures, find the most likely number of lower energy states that are occupied. Find the mean energy.

3) Suppose that photons satisfied Fermi-Dirac statistics instead of Bose-Einstein statistics. Find the mean energy for a black body as a function of temperature $T$. What would be the measured values of Stefan’s constant and the Wien constant if this were the case? What would have been the Rayleigh-Jeans law if this were the case?

4) The universe is diffused with microwave radiation that has an energy density distribution of a blackbody with temperature 2.7 K. Its present radius is $14 \times 10^9$ light-years across. Compute the entropy of the universe due to this microwave radiation. What are the available number of states? The universe is expanding and as it expands it cools down. Using the second law of thermodynamics, find how the temperature depends on the radius of the universe.

5) Consider a “solar sail” of cross section 1 m$^2$ and mass 1 kg. The sail is aligned such that it is facing the sun and is perfectly absorbing of radiation across all frequencies. Assume that the sail starts off at the planet mercury, a distance $60 \times 10^6$ km from the sun, with zero velocity. How long does it take to travel to the earth, a distance $150 \times 10^6$ km from the sun. In this problem, you can ignore the effect of gravity. The surface temperature of the sun is 6000 K and its radius is $7 \times 10^5$ km.