1) Consider $N$ spinless particles obeying Bose-Einstein statistics. Instead of the particles being in a box, assume that they are trapped in a shallow potential of the form $V(r) = \frac{1}{2}m\omega^2 r^2$.

a) Find the condensation temperature as a function of $N$ and $\omega$. In order to do this problem, you should convert the sum over oscillator states to an integral. You need to take into account the degeneracy of the states for a given energy $E = \hbar \omega (n + 1/2)$.

b) Assuming that there are $10^8$ trapped sodium atoms, what is the frequency $\omega$ if $T_c = 10^{-6}$ K.

c) Find the specific heat as a function of $T$ below the condensation temperature.

2) Consider a gas of spinless particles obeying Bose-Einstein statistics in two dimensions, confined to a box. Show that Bose-Einstein condensation does not occur.

3) Same as in (2), but now assume that the atoms are trapped by a two-dimensional potential of the form $V(r) = \frac{1}{2}m\omega^2 r^2$. Show that Bose-Einstein condensation can occur.

4) Show that the chemical potential $\mu$, for $N$ spinless particles obeying Bose-Einstein statistics behaves as $\mu \sim -k(T - T_c)^2/T_c^2$, and so the heat capacity has the form of figure 11.9 in Mandl.

5) 11.8 in Mandl.

6) 11.10 in Mandl.