Statistical Mechanics, Problems for Tutorial No. 4

The following problems from Mandl: 4.1, 4.2, 4.3

1) The tension force of a stretched plastic rod of length $L$ and temperature $T$ is given by

$$ F = a T^2 (L - L_0) $$

where $a$ is a constant. When $L = L_0$ the heat capacity is $C_L = bT$ where $b$ is a constant.

a) Write down a thermodynamic relation between $dS$, $dL$ and $dE$.

b) Find $(\partial S/\partial L)_T$.

c) Given $S(T_0, L_0)$, find $S(T, L)$ for any other $T$ and $L$.

d) Starting at $T = T_i$ and $L = L_i$ and one stretches the thermally insulated rod quasistatically, find $T_f$ when the rod is stretched to $L_f$.

e) Find the heat capacity $C_L(L, T)$ when the rod’s length is $L$ and not $L_0$.

2) Consider a system distributed over states $r$ with an arbitrary probability distribution $p_r$ and let the entropy be defined by

$$ S = -k \sum_r p_r \ln p_r $$

where

$$ \sum_r p_r = 1 $$

We want to compare this to the canonical distribution where

$$ p_r^{(0)} = \frac{e^{-\beta E_r}}{Z} \quad \quad Z = \sum_r e^{-\beta E_r} \quad \quad S_0 = -k \sum_r p_r^{(0)} \ln p_r^{(0)} $$

If we set the mean energies to be the same for the two distributions

$$ \sum_r p_r^{(0)} E_r = \sum_r p_r E_r = \bar{E}, $$

show that

$$ S - S_0 = k \sum_r p_r \ln \frac{p_r^{(0)}}{p_r}, $$

and that $S_0 \geq S$. 