Semantic Value

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Abstract: Semantic values are entities assigned to expressions by theories in order to account for semantic features of languages, such as truth conditions and inferential connections. Semantic values can typically be located in a type hierarchy based on truth values and objects of reference, or (in the case of intensional languages), truth values at an index and objects of reference at an index. Appeal to semantic values is also frequently associated with a desire for a compositional account of the semantic behavior of complex expressions.

A total theory of linguistic understanding is often taken to require three sub-theories: a syntactic theory, a semantic theory, and a pragmatic theory. The semantic theory occupies an intermediary role – it takes as input structures generated by the syntax, assigns to those structures meanings, and then passes those meanings on to the pragmatics, which characterizes the conversational
impact of those meanings. Semantic theories thus seek to explain phenomena such as truth conditions of and inferential relations among sentences/utterances, anaphoric relations among terms, and ambiguity and incoherence of expressions.

One way in which a semantic theory can provide the required explanations is by associating each expression provided by the syntax with a particular entity called its semantic value. These semantic values then serve both to ground the desired semantic explanations and to provide the pragmatic theory with input material on which to operate. While almost any approach to semantic theorizing can be reified into a theory of semantic value - a Davidsonian truth-theoretic account, for example, can associate atomic expressions with axioms and complex expressions with derivations; a translational theory can associate expressions with expressions in the target language of translation - setting semantic theory in the context of semantic value is most typical of an approach to semantics running through Frege, Carnap, and Montague. This discussion begins with the relatively simple theory of semantic values found in Frege, and progresses through various complications of that basic formula. First, the Fregean framework is extended to a full semantic type hierarchy. Second, the Fregean use of truth as the foundational semantic value is expanded to a notion of indexed truth suitable for intensional and context-sensitive languages. Finally, dynamic semantic theories are considered, which diverge from traditional semantic theories by using qualities other than truth to construct semantic values.

Writing in 1891, Frege in “Function and Concept” considers the function $x^2 = 1$ formed by abstracting from an expression such as $(−1)^2 = 1$. Observing that the result of replacing $x$ by various arguments is sometimes a true equation and
sometimes a false equation, he says:

I now say: ‘the value of our function is a truth-value [Wahrheitswert],’

and distinguish between the truth-values of what is true and what
is false. I call the first, for short, the True; and the second, the False.

(137)

Frege’s shift from the adjectives ‘true’ and ‘false’ to the neologistic nominals
‘the True’ and ‘the False’ marks the beginning of semantic theorizing grounded
in semantic values. ‘Wahrheitswert’ then crosses into English as ‘truth-value’
in Russell’s commentary on Frege in 1903 in *The Principles of Mathematics*, and
the contemporary ‘semantic value’ appears to derive from this usage.

Frege’s semantic theory associates with each expression in the language a re-
ferent [Bedeutung]. The various referents are then the semantic values of the
expressions in the language. Two central theses governing Frege’s conception
of semantic value can be identified:

1. The two fundamental types of semantic values are truth-values and ob-
jects (the referents of singular terms). Other semantic values are derived
from the fundamental types through the construction of functions. Call
this the *categorical principle*. Frege recognizes the derived categories of
(a) first-level n-ary functions from objects and truth values to objects or
truth values (paradigmatically, predicates and truth-functional connect-
ives) and (b) second-order n-ary functions from first-order n-ary func-
tions to objects or truth values (paradigmatically, quantifiers and definite
descriptors).

2. Semantic values of complex expressions are derived from semantic values
of their composite expressions, typically (although not necessarily) via
A standard semantic theory for quantified first-order logic can be formulated in Fregean terms by assigning to predicates the characteristic functions of their extensions, to connectives functions from truth values to truth values, and to quantifiers functions from predicate-assigned functions to truth-values. The resulting theory will, as desired, account for truth conditions of and inferential connections between sentences. Thus, for example, semantic values can be assigned as follows:

- The predicate “F”, interpreted as *is French*, is assigned the function $f$ from objects to truth values such that $f(x)$ is the True if and only if $x$ is French.

- The connective “∨”, interpreted as *or*, is assigned the function $g$ from pairs of truth value to truth values such that $g(x, y)$ is the True if and only if at least one of $x$ and $y$ is the True. The connective “¬”, interpreted as *not*, is assigned the function $h$ from truth values to truth values such that $h(x)$ is the True if and only if $x$ is the False.

- The quantifier “∃”, interpreted as *something*, is assigned the function $j$ such that $j(x)$ is the True if and only if $x$ is a function from objects to truth values which does not map every object to the False.

From these semantic values, it can be determined that $\exists xFx$ is true if and only if something is French, and that $\exists x(Fx \lor \neg Fx)$ is a logical truth.

The basic Fregean framework of semantic values coupled with the categorical and compositional principles then generalizes in three significant directions in later semantic work. First, the space of semantic values is extended to a full type hierarchy based on truth values and objects. The roots of this extension can be
seen in Tarski’s 1933 paper “The Concept of Truth in Formalized Languages”, but the full implementation of the thought in semantic theorizing about natural languages occurs with Montague’s “The Proper Treatment of Quantification in Ordinary English” and Lewis’ “Generalized Semantics”. Introductory treatments can be found in Heim and Kratzer’s *Semantics and Generative Grammar* and in Dowty, Wall, and Peters’ *Introduction to Montague Semantics*.

In Montagovian semantics, a categorical grammar is linked with a type-hierarchy of semantic values. The categorical grammar takes a small collection of primitive syntactic categories. Ajdukiewicz’s work introducing categorical grammars in “Syntactic Connexion” uses $N$ (name) and $S$ (sentence) as the primitive categories. Derived categories are then defined as follows:

• $\langle \alpha/\beta \rangle$ = the category of expressions combining with a $\beta$-category expression to form an $\alpha$-category expression.

Thus:

• Intransitive verbs can be of category $\langle S/N \rangle$. “Snores” can combine with an expression of category $N$ (“Socrates”) to form an expression of category $S$ (“Socrates snores”).

• Nouns can also be of category $\langle S/N \rangle$. “Linguist” can combine with an expression of category $N$ (“Socrates”) to form an expression of category $S$ (“Socrates is a linguist”). Placement of nouns in $\langle S/N \rangle$ is somewhat syntactically forced and is influenced by the treatment of nouns as predicates in first-order logic; other versions of categorical grammar contain a third primitive category $T$ of nouns.

• Modal operators can be of category $\langle S/S \rangle$. “Necessarily” can combine with an expression of category $S$ (“Aristotle is fond of dogs”) to form an
expression of category $S$ ("Necessarily, Aristotle is fond of dogs").

- Quantified noun phrases can be of category $< S/ < S/N >>$. "Some linguist" can combine with an expression of category $< S/N >$ ("snores") to form an expression of category $S$ ("Some linguist snores").

- Determiners can be of category $<< S/ < S/N >> / < S/N >>$. "Some" can combine with an expression of category $< S/N >$ ("linguist") to form an expression of category $< S/ < S/N >>$ which in turn combines with an expression of category $< S/N >$ ("snores") to form an expression of category $S$ ("Some linguist snores").

The type-hierarchy of semantic values similarly takes a small collection of primitive semantic categories. Following the Fregean tradition (and setting aside issues of intensionality to be raised below), the primitive categories can be $t$ (the set of truth values) and $e$ (the set of objects). Derived categories are then defined as follows:

- $< \tau, \pi > = \{\text{the set of functions from category } \pi \text{ to category } \tau\}$.

If we then associate each syntactic category with a semantic category via a mapping $\llbracket \cdot \rrbracket$ such that:

1. $\llbracket N \rrbracket = e$
2. $\llbracket S \rrbracket = t$
3. $\llbracket < \alpha/\beta > \rrbracket = \llbracket \alpha \rrbracket / \llbracket \beta \rrbracket$

we can then obtain a semantic theory in which the semantic value of any complex expression results from the functional application of the semantic value of one of its immediate constituents to the semantic value of the other of its constituents. Both nouns and intransitive verbs, for example, are assigned
functions from objects to truth values. Those functions can then be treated as characteristic functions determining predicate extensions.

This broadly Montagovian framework provides a fertile setting for much work in formal semantics. Barwise and Cooper’s work on generalized quantifiers, for example, fits naturally into such an approach. The categorical grammar yields the categories of $<< S/ < S/N >> / < S/N >>$ for determiners and of $< S/ < S/N >>$ for quantified noun phrases. The type hierarchy of semantic values then assigns values of type $<< t/ < t/e >> / < t/e >>$ to determiners and of type $< t/ < t/e >>$ to quantified noun phrases. “All philosophers”, for example, is assigned a function from extensions (in the form of characteristic functions from objects to truth values) to truth values. The appropriate function is that which maps to the True sets which have the set of philosophers as a subset, and to the False other sets. The determiner “all” then receives as semantic value a function from $< t/e >$ values (predicate extensions) to quantified noun phrase values $< t/ < t/e >>$. The appropriate function for “all” is that which maps each set $X$ of objects to the function which maps a set $Y$ to the True if and only if $X \subseteq Y$. The type hierarchy yields a natural space of all possible quantifiers, allowing the location of semantic universals governing quantifiers in natural language, of the sort Barwise and Cooper set out. Thus monotone increasing quantifiers, such as “some linguist” and “every mathematician”, are assigned functions $f$ such that, for any extensions $X$ and $Y$, if $f(X)$ is the True and $X \subseteq Y$, then $f(Y)$ is also the True. Monotone increasing quantifiers thus support inferences of the form:

- Some linguist owns a red car. Therefore, some linguist owns a car.

in which the predicate of the conclusion is more restrictive than that of the premise. Monotone decreasing quantifiers, on the other hand, are assigned func-
tions \( f \) such that, for any extensions \( X \) and \( Y \), if \( f(X) \) is the True and \( Y \subseteq X \), then \( f(Y) \) is also the True. Monotone decreasing quantifiers such as “no archaeologist” and “few physicists” support inferences of the form:

- Few physicists own a red car. Therefore, few physicists own a car.

in which the predicate of the conclusion is less restrictive than that of the premise. Not all quantifiers are monotone in either direction, as examples like “an even number of chemists” show – Barwise and Cooper thus hypothesize that all simple natural language quantifiers are conjunctions of monotone quantifiers.

Many complications of the simple type hierarchy of Montagovian semantics can now be investigated. Partee and Rooth’s “Generalized Conjunction and Type Ambiguity”, for example, argues that expressions cannot be assigned a single stable semantic category. In the sentence:

- John caught and ate a fish

the transitive verbs “caught” and “ate” need to be assigned the category \(< < t/e > /e >\). If each transitive verb is assigned a function from entities (\( e \)) to intransitive verb extensions (\(< t/e >\)), then their conjunction will be assigned the function which maps any entity \( x \) to the intransitive verb which assigns the True to any entity which caught and ate \( x \). This function can then combine with the entity assigned to “John” to create an appropriate truth value for the whole sentence. However, this category assignment for transitive verbs fails in other examples such as:

- John needed and bought a new coat.

The truth value of this sentence does not depend on a function which maps entities \( x \) to intransitive verbs assigning the True to entities which needed and
bought $x$, since we do not here require that John needed and bought any one particular entity. Instead, we require that John needed and bought one type of entity. Thus the transitive verbs “needed” and “caught” must be assigned the category $<< t/e > / < t/ < t/e >>>$. By assigning to each transitive verb a function from nouns ($< t/e >$) to truth values, their conjunction will then be assigned the function which assigns the True to any object which needed and bought that type of object. Partee and Rooth thus suggest that the semantic value assigned to an expression can undergo type raising which moves it from a simpler to a more complex type to fit constructions such as these.

The second generalization of the core Fregean framework is the move to truth at a specification point. The first instance of this is the intensionalization of Frege’s extensional logic. The roots of intensionalization can already be seen in Frege’s separation of sense and reference in “On Sense and Reference”, but the full picture emerges in Carnap’s *Meaning and Necessity*. Carnap introduces state descriptions – negation-complete sets of atomic sentences equivalent to possible worlds – and defines truth in a state description. He then assigns to every expression both an extension and an intension. Its extension is its semantic value in the Fregean truth value/object-based type hierarchy, and its intension is its semantic value in a parallel hierarchy based on functions from state descriptions to truth values and functions from state descriptions to objects. Given a designation of an extensionally privileged state description (the “actual world”), the extensional type hierarchy can be subsumed into the intensional. The intensional type hierarchy is connected conceptually to the Fregean type hierarchy via an understanding of truth at a world in terms of truth simpliciter (what would be true, were the world a certain way). This conceptual connection allows the intensional semantics still to address questions of truth
conditions and inferential connections.

The basic insight of the intensionalizing move is that a finer-grained semantics can be achieved by replacing truth values simpliciter in the type hierarchy with indexed truth values, in the form of functions from some set of indexes to truth values. The result is a semantic theory equipped for non-truth-functional operators such as modalities. A similar approach can thus accommodate tense by moving to truth at a time (or at a time/world pair). The same insight is employed in adapting the Fregean framework to context-sensitive expressions.

Kaplan, in “Demonstratives”, gives a semantic theory in which expressions are assigned intensions (both modal and temporal) with respect to a context, where a context provides information at least about utterance speaker, spatial and temporal location, and world. Kaplan’s system thus uses a form of double-indexing - the background type hierarchy is grounded on functions from pairs of indices <world, time>, context> to truth values and objects. Thus the sentence:

- I am a philosopher

can be assigned a truth value only relative first to a context which determines the referent of the indexical “I” and second to a world-time pair that determines the properties of the contextually-provided referent. Kamp’s approach to tense in “Formal Properties of Now” uses a similar system of double indexing, and Stalnaker in “Assertion” gives a streamlined version of the double-indexing approach, based on Segerberg’s two-dimensional modal semantics, in which both indices are simply worlds.

Context-sensitive semantic theories thus extend the Fregean framework further in the direction begun by intensional semantics, by extending the indexing of
truth at the foundation of the type hierarchy. The introduction of double-indexing gives rise to a novel issue in interpretation. The single-indexing of intensional semantics typically comes with a collection, in the analyzed language, of intensional operators which act on the indexing position – thus the expression “necessarily” binds the world index in the semantic value of expressions it governs, causing them to be evaluated at every possible world. As a consequence, the new foundational semantic category of truth-at-an-index can be reductively understood via the simple truth of some claim containing an intensional operator. For example, a claim \( \varphi \)'s being true at a world \( w \) can be reduced to the intensionally modified claim “If it were the case that \( w^* \), it would be the case that \( \varphi \)'s being simply true (where \( w^* \) is a canonical description of world \( w \)). However, in double-indexed semantic theories, the second indexed position is typically not subject to semantic interaction with intensional operators of the language. In a sentence such as:

- I am always here.

the intensional operator “always” controls the first index, governing the temporal evaluation of the verb. It does not, however, govern the second index, determining the contextual assignment of semantic values to “I” and “here”. Double-indexing thus means that truth at an index can no longer be understood in terms of what would be (or was, or will be) true simpliciter. It requires instead a novel understanding of the elements of the type hierarchy. The result is the idea that semantic value characterized at a certain level (“utterance meaning”) varies based on context of use, so that the same expression can have different semantic contents in different contexts of use.

The third and final generalization of the Fregean framework is the move to dynamic semantics. Dynamic semantics can be thought of as the other side of the
coin of context sensitivity, investigating the way that expressions affect, as well as are affected by, context. Dynamic semantic theories have flourished over the last 25 years, taking such forms as discourse representation theory, file change semantics, dynamic predicate logic, and update semantics. When placed in a compositional form, one distinctive characteristic of dynamic semantic theories is a move away from indexed truth to a non-truth-based fundamental semantic value. Overviews of major topics in dynamic semantics can be found in Gamut’s Logic, Language, and Meaning and in Kamp and Reyle’s From Discourse to Logic.

Groenendijk and Stokhof’s dynamic predicate logic, for example, makes dynamic the semantic analysis of sentences of first-order logic using sets of satisfying assignments to variables (where truth is equated with satisfaction by all assignments). In dynamic logic, a sentence is associated not with a static set of assignments, but with a function from input assignments to output assignments. An existential claim of the form “∃xFx” pairs an incoming assignments g with an outgoing assignment h if and only if h satisfies Fx and there is some assignment g ∈ G such that g and h differ at most in the x position. Quantifiers thus have unrestricted rightward scope – by passing on assignments whose x positions satisfy the existentially quantified matrix, future reference to and description of these objects is then possible. This unrestricted rightward scope allows dynamic predicate logic to model the behaviour of cross-clausal anaphora on indefinite noun phrases, as in:

- A man walked in the park. He wore a hat.

The dynamic move to non-truth-based semantic values is more radical than the intensionalizing move to semantic values based on truth at an index. One consequence is that associated semantic concepts require redefinition. Equiva-
lence, for example, can no longer be understood as sameness of truth value in all models, in the absence of truth-based semantic values. Dynamic predicate logic allows multiple notions of equivalence, the stronger of which is identity of behaviour in input-output conditions:

- $\phi \simeq \psi$ if $\phi$ pairs an input assignment $g$ with an output assignment $h$ if and only if $\psi$ does as well.

$\exists x Fx \land Gx$ is not equivalent to $Gx \land \exists x Fx$ in this sense, since an assignment $g$ whose $x$ position does not satisfy $Gx$ will produce no output when input to $Gx \land \exists Fx$, but can produce an output when input to $\exists x Fx \land Gx$, since in the latter case the $x$ value can be ‘reset’ to an object satisfying $F$ and $G$, if there is such an object. Similarly, dynamic predicate introduces a new consequence relation:

- $\phi \models \psi$ if and only if for any model $M$ and any assignment $g$ in $M$, if the input of $g$ to $\phi$ produces some output assignment $h$, then the input of $h$ to $\psi$ produces some further output assignment $k$.

The resulting consequence relation differs from classical consequence relations in many structural features. Idempotence fails — we do not have $Gx \land \exists Fx \models Gx \land \exists Fx$, since the output assignment, with its reset value in the $x$ position, need not satisfy $Gx$. Transitivity similarly fails, since $\neg\neg\exists x Fx \models \exists Fx$ and $\exists Fx \models Fx$, but $\neg\neg\exists x Fx \not\models Fx$, since negation blocks dynamic effects.

References


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**Biography**

**Josh Dever** is Assistant Professor of Philosophy at the University of Texas. He received his Ph.D. in 1998 at the University of California at Berkeley. He works in the philosophy of language and philosophical logic, and has published on the principle of compositionality and the semantics of referential expressions.